How-To: Matrix Multiplication

To multiply two matrices, the following must be true:

$$\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

Dimensions:    3 X 2      2 X 3

$$\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

Dimensions:    3 X 2      2 X 3

These dimensions MUST match to multiply    The answer matrix will have these dimensions

So $\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix}$ some 3 X 3 matrix

To find the elements of this new matrix, you select a row from the first matrix and a column from the second.

$$\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \quad \rightarrow \begin{bmatrix} \blacksquare & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix}$$ This element is in Row 1, Column 1

MULTIPLY each element from the given row with the corresponding element from the given column and ADD the results.

$$\begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + -1 \times -2 & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 2+2 & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 4 & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix}$$

For R1, C2
$$\begin{bmatrix} \circ & \blacksquare & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} \quad \begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \times 2 + (-1) \times 3 & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 4 & 4-3 & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 4 & 1 & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix}$$

For R2, C1
$$\begin{bmatrix} \circ & \circ & \circ \\ \blacksquare & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} \quad \begin{bmatrix} 2 & -1 \\ 0 & 8 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 & \circ \\ 0 \times 1 + 8 \times -2 & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 4 & 1 & \circ \\ 0 - 16 & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix} = \begin{bmatrix} 4 & 1 & \circ \\ -16 & \circ & \circ \\ \circ & \circ & \circ \end{bmatrix}$$

Repeat process for the remaining 6 spots. The final result is: $\begin{bmatrix} 4 & 1 & -1 \\ -16 & 24 & 8 \\ -2 & 17 & 3 \end{bmatrix}$

Ex 2:      $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} = [1 \times 2 + 0 \times 2 + 3 \times 5] = [2 + 0 + 15] = [17]$

Dimensions:   1 X 3    3 X 1    Can be multiplied. The answer is a 1 X 1 Matrix

Ex 3:      $\begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 & 0 \\ -5 & 4 & 2 \end{bmatrix}$

Dimensions:   3 X 1    2 X 3    Can NOT be multiplied. Write "Not Possible"

Note: Matrix multiplication is not commutative, meaning $A \times B$ is not the same as $B \times A$